4.11 Proving the Side-Splitter Theorems

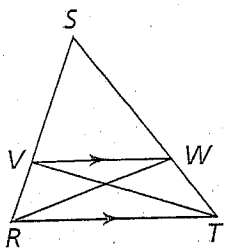
Recall what the first part of the Parallel Side-Splitter Theorem says:

> If a segment with endpoints on two sides of a triangle is parallel to the third side of the triangle, then it splits the sides it intersects proportionally.

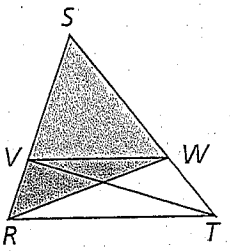
To prove this theorem, you can show that, in the figure below, if $\overline{VW} \parallel \overline{RT}$, then $\frac{SV}{VR} = \frac{SW}{WT}$.

In the figures below, triangles SVW and RVW have the same height. This means that the ratio of their areas is equal to the ratio of their base lengths, SV and VR.

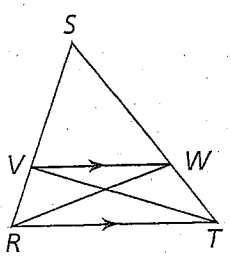
This fact comes from Lesson 4.9.



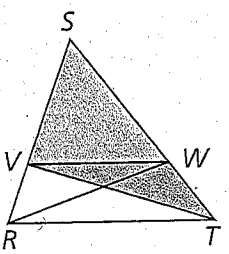
$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} = \frac{SV}{VR}$$



In the figures below, triangles SVW and TVW have the same height. Thus, the ratio of their areas is equal to the ratio of their base lengths, SW and WT.



$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)} = \frac{SW}{WT}$$



Habits of Mind

Understand the process. A "tidy" proof like this may not necessarily be easy to write. You build it from many notes, sketches, erasures, more sketches, more notes, and plenty of talking to yourself!

You have two fractions with the same numerator,

$$\frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} \text{ and } \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)}.$$

The denominators are not the same, but are they equal? Recall Problem 1 from Lesson 4.9 Getting Started. Triangles RVW and TVW share the same base $\overline{VW}$. They have the same height since $\overline{VW}$ is parallel to $\overline{RT}$. So they have the same area.

You can combine all of these results to draw a conclusion about SV, VR, SW, and WT.

$$\frac{SV}{VR} = \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle RVW)} = \frac{\text{area}(\triangle SVW)}{\text{area}(\triangle TVW)} = \frac{SW}{WT}$$

For You to Do

1. Using what has been outlined in this lesson, write a complete proof of the first part of the Parallel Side-Splitter Theorem.

Exercises *Practicing Habits of Mind*

Check Your Understanding

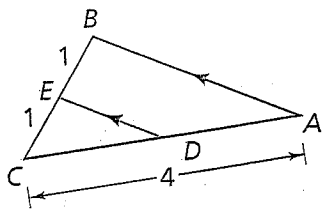
1. Recall the Proportional Side-Splitter Theorem:

   If a segment with endpoints on two sides of a triangle splits those sides proportionally, then the segment is parallel to the third side.

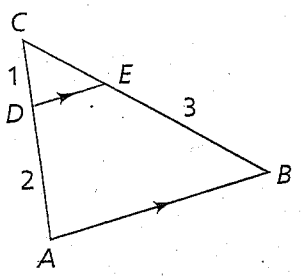
   This time, the proof is up to you. Use the same setup that you used to prove the Parallel Side-Splitter Theorem. Write your proof so that sometone else can follow it.

2. In the diagrams below, $\overline{AB}$ is parallel to $\overline{DE}$. Find as many lengths as you can.

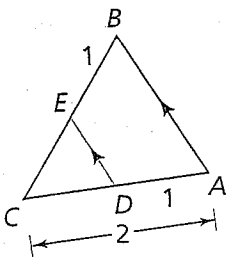
   a.

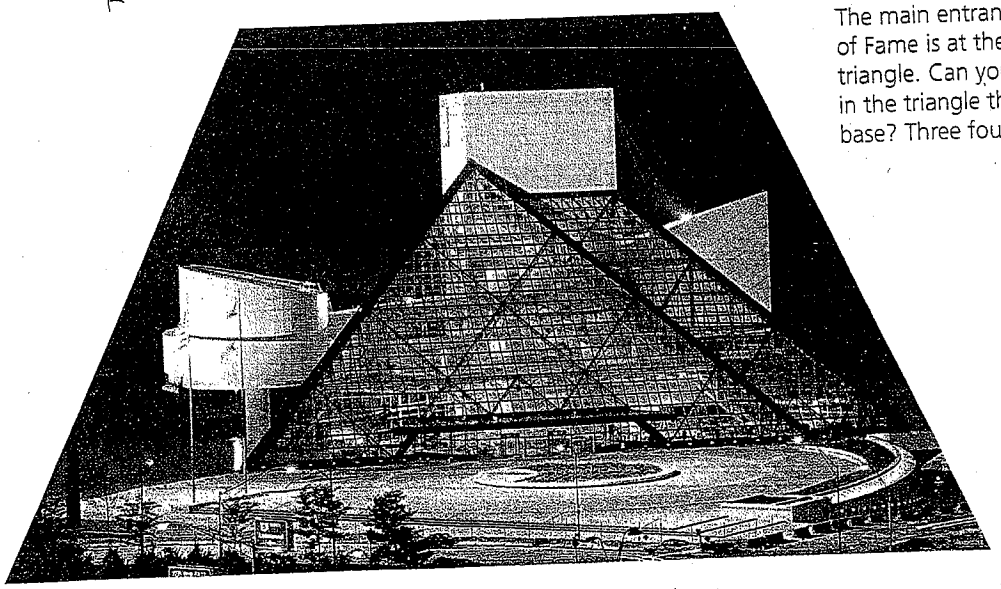


   b.



   c.





The main entrance to the Rock and Roll Hall of Fame is at the base of this glass-and-steel triangle. Can you find horizontal steel ribbing in the triangle that is half the length of the base? Three fourths the length of the base?